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A generalized stochastic frontier production function in power loom industries for prediction of firm-level technical efficiencies

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ABSTRACT

A production function in stochastic frontier is constructed for a set of sample firms. The disturbances are appear to be in two different terms, one is the traditional error and the other lies with a non-negative assorted variable associated with the Technical talent of the firm. By assuming the acceptable company's impact as time invariant with a common incomplete distribution, we have obtained the best prediction for the firm-effect random variable and also the Technical talent of any firm given the cost of troubles in the model.

KEY WORDS: Frontier Production Function, Technical Efficiency, Random variable, Standard Deviation, Half-Normal distribution.

1. INTRODUCTION

A Production function is a technical relationship between inputs and output, which is usually, specified as

$$Y = f(X) \exp(w) \quad \text{----------------------- (1)}$$

Where Y, a (nx1) vector of observed output, X, a (nxk) matrix of inputs, and exp (w), the stochastic error term.

A frontier production function is one that relates the maximum feasible output corresponding to the best practiced technique among the given producers with their inputs denoted as

$$Y^* = f(X) \exp(w) \quad \text{--------------------------(2)}$$

Y^*- maximum feasible output

A major limitation of the frontier production function is its assumption of deterministic relationship which avoids the real validity that a firm's performances may be altered by factors entirely outside its control as well as by factors under its control. The former is the collective effect of external forces shocks both favourable and unfavorable and the latter is due to inefficiency in use of technology. Hence the two sources of errors need to be separated to realise the real effect of inefficiency. This is the idea behind the frontier production function.

In stochastic frontier model the error is made of two parts. A systematic component which allows assorted variation of the frontier throughout the firms and comprises effect of statistical noise and random forces external control of the firms. A one sided component absorbs the effect of inefficiency relative to the stochastic frontier. The model is

$$Y = f(X) \exp(v-u) \quad \text{------------------------------- (3)}$$

Here the stochastic frontier is f(X) exp (V) where v has symmetric distribution to seize the random effect measurement errors and external forces which causes the where about of the deterministic kernel f(X) to vary across the firms. Technical inefficiency relative to the stochastic production frontier is then captured by one-sided error component exp (-u), u>o. This condition ensures that all observation lie beneath the stochastic production frontier.

In (3) v_{ij} are random variables which are i.i.d. $N\left(o,\sigma_v^{\,2}\right)$ independent of the u_i-random variables, which are i.i.d non-negative random variables, defined by the truncation (at zero) of the $N\left(\mu,\sigma^2\right)$ distribution. If error is assumed as exp (v+u) then u is appears to be definitely negative.

Firm Technical efficiency: The technical efficiency of an approved firm is defined as the ratio of its mean production (in primitive units), given its realized firm effect, to the corresponding mean production if the firm impact is zero. Thus if TEi stands for the technical efficiency of the i^{th} firm

$$Te_i = \frac{E\left(\dfrac{Y_{it}}{u_i}, X_{it}, t = 1,2..........\right)}{E\left(\dfrac{Y_{it}}{u_i} = 0, X_{it}, t = 1,2,.....\right)} \text{----------------(4)}$$

Where Y_{it} denote the value of production for the i^{th} firm in the t^{th} time period.

Obviously $0 \le Te_i \le 1$.Te_i=0.85 implies that the firm realizes on the average 85 percent of the production possible for a fully efficient firm having comparable input values.

If the form of the frontier production function is

$$Y_{it} = X_{it}\beta + e_{it} \text{-------------------------------------- (5)}$$

And e_{it}=v_{it}-u_i where Y_{it} is the logarithm of the output of the i^{th} firm in the period t, U_i is assumed to be strictly positive

Te_i=exp (-u_i) ------------------------------------ (6)

Schmidt and sickles (1984) took into effect many methods of predicting individual; firm effects (and hence, technical efficiencies) approved that data were present on model firms. Waldman (1984) found out the qualities of a prediction for firm technical efficiencies proposed by Jondrow (1982) and other available predictors.

In this article we submit a generalization of a few outcomes given by Jondrow (1982), under the assumption that data on model firms are available and that a more common supply for firm effects, suggested by Stevenson (1980), apply for the stochastic frontier production function. Stevenson has suggested truncated normal and normal distributions for U_i and V_i respectively. Thus

$$F(u_i) = \frac{\sqrt{\frac{2}{\pi}}}{\sigma_u} \exp\left(-\frac{u^2}{2\sigma_u{}^2}\right) \quad if\ u_i \le 0 \text{ -----------(7)}$$

$$= 0, Otherwise$$

$$and \quad f(v_i) = \frac{1}{2\pi\sigma v} \exp\left(-\frac{v_i{}^2}{2\sigma v^2}\right) \quad -\infty < v_i < \infty$$

The supposed function of Y is equal to the product of density function of each Y_i which is the density function of (u_i+v_i) which by convolution is

$$f\left(u_i + v_i\right) = f(y_i) = \sigma\sqrt{\frac{2}{\pi}} \exp\left(\frac{-w_i^2}{2\sigma^2}\right)(1 - F\left(\frac{w_i}{\sigma}\sqrt{\frac{r}{1-r}}\right) \text{ ----------------- (8)}$$

Where F(*) is the cumulative distribution function of the standard normal variate,

$$\sigma^2 = \sigma_u{}^2 + \sigma_v{}^2$$

$$\gamma = \frac{\sigma_u{}^2}{\sigma_u{}^2 + \sigma_v{}^2}$$

$$and \ u_i + v_i = w_i = Y_i - \sum \beta_j X_{ij}$$

When $\sigma_v{}^2$ is zero then r=1 and U_i is the most important mistake which indicates that the low level of output is because of the shortage of technical efficiency. Contrary to which σu^2 is zero implies r=0 which means that the low level in output is due to external factors not control.

The likelihood function for the sample firms (Y_1, Y_2,…….., Yn) is

$$L(Y,\theta) = \prod_{i=1}^{n} \frac{1}{\sigma}\sqrt{\frac{2}{\pi}}\left(1 - F\left\{\frac{Y_i - \sum \beta_j X_{ij}}{\sigma}\sqrt{\frac{r}{1-r}}\right\}\right).\exp\left(-\frac{1}{2}\frac{\left(Y_i - \sum \beta_j X_{ij}\right)^2}{\sigma^2}\right) \text{ --------- (9)}$$

Where θ is the parameter which contains the elements $\left(\beta_0, \beta, \sigma^2, r\right)$

Empirical application: Power loom Industry in Salem district, Tamilnadu is presently structured according to regulations and requirements within this district. Two types of cloths are produced one is pure cotton and another is 100 percent synthetic. We try to test if the mean technical efficiencies in the two types are equal and to predict the individual technical efficiencies.

The frontier production function specified for this is

$$Y_{it} = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \beta_3 X_{3t} + v_{it} - u_i \text{ ------------------------ (10)}$$

Where the subscript i (i=1,2,………..N) refers to the i^{th} sample and the subscript t(t=1,2,3) refers to the t^{th} year:Y implies the logarithm of the total gross returns including receipts from subsidiary products (net of levies, freight and handling charges etc.,):X_1 denotes the logarithm of the value of the total labor(in work weeks) which includes operator labour,other family labour, partner labour and hired lobour. X_1 denotes the logarithm of the cost of all inputs and X_3 denotes the logarithm of the total cost of the capital which involves the replacement cost of structures, equipment and depreciation.

The variable of the model (10) are expressed in value terms, rather than physical units, because the latter were not available exactly. However, the cost and price structure in the two firms are similar. The random variable V_{it} and U_i in the model (10) are understood to have the properties specified in (7).

The ordinary Least squares estimations (OLS) of β_1, β_2, β_3 in (10) are unbiased. Since the mean of the assorted variable U_i is positive, the OLS of β_1 in negatively biased. The OLS estimate of intercept, slope and variance parameter for the production function of the firms are presented in table.1. Given that (10) in the true model, then the estimated standard deviations for the estimators of the parameters are not the correct ones according to the OLS method.

The OLS estimates are directly used as initial estimates or used to obtain initial estimates, for the Davidon-Fletcher-Powell method of approximating the maximum likelihood (ML) estimates for the parameters of the frontier model (10) for the power loom industries combined. Initial calculation for the ratio-variance parameter were considered between 0.1 and 0.9. An array of initial estimates were considered for the mean and the corresponding initial estimates were obtained for the intercept and the total variance.

$$\gamma = \frac{\sigma_u^{\ 2}}{\sigma_u^{\ 2} + \sigma_v^{\ 2}}$$

$$\sigma^2 = \left(\sigma_u^{\ 2} + \sigma_v^{\ 2}\right).$$

The supposed maximum-likelihood estimate for the parameters of the frontier model for the two types of firms and the combined one are presented in Table 1.

Table.1. parameters of the frontier model for the two types of firms and the combined one

Industry	**Variable**			
	Intercept	**Labour**	**Input**	**Capital**
Pure cotton (N=40)				
O.L.S	-0.71 (0.59)	0.144 (0.036)	0.384 (0.034)	0.670 (0.055)
M.L	-0.47 (0.23)	0.0989 (0.013)	0.3218 (0.0085)	0.0716 (0.021)
M.L	-0.337 (0.045)	0.0735 (0.0036)	0.3582 (0.0029)	0.7119 (0.0039)
Synthetic (N=62)				
OLS(R^2=0.74)	0.72 (0.49)	0.62 (0.077)	0.191 (0.037)	0.728 (0.045)
M.L	1.257 (0.024)	0.0668 (0.0044)	0.0889 (0.0015)	0.7993 (0.0019)
M.L	1.597 (0.009)	0.0257 (0.0019)	0.0799 (62110^5)	($85/10^5$)
Both combined (102)				
OLS(R^2=0.77)	0.12 (0.32)	0.146 (0.041)	0.248 (0.018)	0.689 (0.035)
ML	1.08 (0.18)	0.029 (0.0031)	0.1776 (0.0016)	0.7853 (0.0031)
ML	1.01 (0.008	0.376 (0.0011)	0.169 (0.006)	0.787 (0.0009)

Variance Parameter					
				Log-Likelihood	**MeanTechnical efficiency**
Pure cotton (N=40)					
O.L.S	0.089	0.0	0.0	23.56	-
M.L	0.108 ($31/10^6$)	0.419 (0.057)	0.22 (0.18)	-15.51	0.73 (0.117)
M.L	0.179 ($15/10^6$)	0.587 (0.037)	0.0	-16.87	0.757 (0.016)
Synthetic (N=62)					
OLS (R^2=0.74)	0.118	0.0	0.0	-61.77	-
M.L	0.199 ($12/10^6$)	0.689 (0.019)	0.43 (0.31)	-32.81	0.591 (0.089)
M.L	0.398 ($21/10^6$)	0.888 (0.0098)	0.0	-43.80	0.0649 (0.0014)
Both combined (102)					
OLS (R^2=0.77)	0.116	0.0	0.0	-108.51	-
ML	0.141 ($2/10^5$)	0.558 (0.021)	0.85 (4.00)	-63.89	-
ML	0.359 ($5/10^5$)	0.884 (0.008)	0.0	-79.96	-

The estimates for the standard errors of the maximum-likelihood estimates for the parameters, obtained by the method of Berndt (1974), are presented in parantheses below the maximum-likelihood estimates. M.L. estimates are also presented for the parameters of the frontier production function when the positive random variable U_i has half-normal distribution (i.e. μ=0). Over and above, what are given are estimates of the basic technical efficiencies for the original frontier model (10) and restricted case involving the half-normal distribution.

A joint test on the significance of the random variable U_i in the frontier model (10) is obtained from the generalized-likelihood ratio. If the random variable is absent from the model (i.e.μ=r=0), then the OLS estimators of the remaining parameters of the production function are M.L estimators. Thus, the negative of twice the logarithm of the generalized-likelihood ratio has roughly chi-square distribution with parameter equal to two. The values of the test statistic for cotton and synthetic firms are 16.7 and 63.5 respectively which are highly important. We thus conclude that both parameters of the distribution of the random variable U_i are not zero, and so it is important for explaining the distribution of total returns for the two firms. Further, if the parameter μ has value zero, then twice the negative of the logarithm of the generalized similar ratio for the condensed and unrestricted ($\mu \neq 0$) frontier models has approximately chi-square with parameter equal to one. The value of this statistic are 2.41 and 7.62. These values are significant only at five percent level and so we conclude that the restricted frontier ($\mu=0$) is not an adequate representation for the two power loom industries.

Estimates of the mean technical efficiencies based on the frontier production function (10) indicate that the cotton producing Industry is 77% technically efficient and the synthetic firm is 59.1% efficient.

2. CONCLUSIONS

Our application of stochastic frontier functions to power loom Industry in Salem indicates that the traditional Cobb-Douglas type of production function is adequate in determining the technical efficiencies of firms.

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